Light Scattering Induced by Turbulent Flow

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Abstract

Light scattering induced by turbulent flow in seawater has been studied and the effect of seawater turbulence on the propagation of a collimated light beam has been characterized. Inhomogeneities in the Refractive Index (IRI) of seawater are characteristic of turbulent flows. Our approach is to describe the interaction of light with IRI by solving Maxwell's equations. This set of equations is converted into the parabolized Helmholtz equation in the case of light propagating through water with IRI. We characterize the light scattering within a water parcel by the Volume Scattering Function (VSF). Field measurements of small-angle VSF exhibit a sharp peak which is orders of magnitude greater than that obtained from either laboratory measurements or Mie calculations for suspended particles. This has been postulated in the literature to be turbulence related. Our computer simulations show that the volume scattering function obtained is indeed characterized by an exponential decrease with scattering angle and is in quantitative agreement with *in situ* observations in the case of high temperature variance dissipation, χ. These results are in qualitative agreement with a previous study using the ray tracing technique. It appears that O(1°) is the upper limit of turbulent induced light scattering in the ocean.

1. Introduction

The passage of a coherent electromagnetic beam through a pure medium in turbulence results in a change of light velocity which in turn causes distortion in intensity and phase of the beam. Light propagation through the Earth's atmosphere has been studied extensively and many models have been developed to describe atmospheric turbulence (see for example the review by Strohbehn[1]). Similar studies of the propagation of light in turbulent water are to date very sparse [2]. The quantitative description of light scattering within a water parcel is given by the volume scattering function (VSF) (for example Spinrad [3]). Field measurements of small-angle scattering [4] show that the VSF exhibits a sharp peak which is orders of magnitude greater than that obtained from either laboratory measurements of Spinrad[3] or Mie type calculations for non-turbulent conditions[5]. This effect (*i.e.* peaking of volume scattering function at small angles) has been attributed to scattering by turbulence induced inhomogeneities in the refractive index of sea water (see Yura[6]). One of the main goals of this paper is to verify this conjecture.

Currently, increased use of optical detectors in the marine environment (for example biological oceanography and in underwater imaging and communications) makes quantifying the role of turbulence an important task. Fortunately, only small-scale fluctuations in the refractive index due to the flow field are relevant for light propagation. These small-scale features are only weakly dependent on the large-scale flow and thus are similar for different flow types[7].

The interaction of light with fluid inhomogeneities associated with turbulence has long been utilized in laboratory observations of turbulence (*i.e.*, using the shadowgraph technique)[8]. This technique can also been used *in situ*, as was done to visualize a double-diffusive instability [9].

It is extremely difficult to obtain optically clean water for laboratory experiments of light scatter due only to turbulence; thus we have chosen to do a numerical experiment. Any simplifying assumptions about the structure of turbulent flow fields can lead to unexpected errors[10], and so we have chosen to use a direct numerical simulation. We propagate light through this modeled turbulent field to quantify the interaction of light scatter. Here we present the results of our numerical experiment.

2. Theoretical Background

2.1. Inhomogeneities in the refractive index

Inhomogeneities in the refractive index (IRI) are characteristic of turbulent flows. Under typical oceanic conditions, these inhomogeneities appear to be temperature dominated. In general, however, the real part of the refractive index of seawater varies with changes of temperature, salinity, and pressure (changes in the imaginary part of the refractive index of water are negligibly small compared with the real part[12]). The effect of pressure can be neglected for spatial scales on the order of a meter or smaller. The variance of the IRI can be expressed as the sum of the variance associated with temperature and salinity: $\overline{(n^2)}^{1/2} = \overline{(n_T^2)}^{1/2} + \overline{(n_S^2)}^{1/2}$, where $\overline{()}$ denotes spatial averaging, n is the refractive index, and n_T and n_S are the contributions of temperature and salinity respectively.

In the ocean, background temperature and salinity induce quite different variance in the IRI. For example, data from the coastal region off Oregon (J. Moum, pers. comm.) show that variance of the refractive index due to temperature is much larger than that due to salinity. Thus, only temperature induced IRI will be considered in this study, so hereafter $n = n_T$.

The largest and smallest size of temperature inhomogeneities, and consequently of IRI, can be inferred from the observed oceanic rates of turbulent kinetic energy dissipation, ϵ. In the lowest limit, the spatial scale is approximately given by the Kolmogorov scale $2\pi(\nu^3/\epsilon)^{1/4}$, where ν is the molecular viscosity. Under typical oceanic conditions, dissipation is found to be between 10^{-9} and 10^{-6} W/kg [13]. Given this range of dissipation, the Kolmogorov length typically varies between 10^{-3} and 10^0 m. The contribution of the IRI on scattering is largest for the smallest IRI structures[6]. Therefore in this study we will concentrate on light scattering by IRI structures at the Kolmogorov scale.

2.2. Interaction of IRI with light

The interaction of light with IRI is described by Maxwell's equations. This set of equations, in the case of forward scattered light, can be converted into the parabolized scalar Helmholtz equation[11]. In this equation, if light initialy propagates along the z-axis, we have:

$$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\psi + 2ik\frac{\partial \psi}{\partial z} + k^2\left(\frac{n^2(r)}{n_0^2} - 1\right)\psi = 0 \tag{1}$$

where ψ represents the amplitude and the phase of the electric field, k is the light wavenumber, $n(r)$ is the spatially varying refractive index, and n_0 is the mean refractive index. We obtain the refractive index field from direct numerical simulation (DNS). This equation is derived from the full set of Maxwell's equations and is accurate as long as the propagating light creates small or moderate diffracting angles with respect to the propagation axis[12] .

3. Methods

To confirm the conjecture about importance of the light scattering by turbulence at small angles we have used data from Ruetsch and Maxey [14]. They simulated turbulent flow field data on a 96^3 grid point flow field with an embedded temperature field as a passive scalar. The Prandtl number used, $Pr = 0.5$, is not very realistic since Pr for water is 7. We use these data to model the temperature distribution. Thus here temperature spectra and velocity are similar.

The low Pr means that the smallest temperature structures will be somewhat larger than the structures in velocity. Therefore the presently used DNS data set does not reproduce representatively small temperature scales. This leads to an underestimate of the volume scattering function. For future work, we will use a simulation with $Pr = 7$.

The flow has a microscale Reynolds number of 60 (and 500 at the cube scale). This is small in comparison to typically observed geophysical flows where the Re is at least of order 10^6 in typical oceanographic applications and over scales of m. The disparity between the Re of realistic flows and that of the simulation is fortunately not an important factor for the light scattering process. It can be shown that the largest contribution to light scattering comes from the smallest temperature scales which are locally isotropic[7]. Thus, in order to reproduce light scattering on the turbulent flow we only have to reproduce accurately small scale fluctuations of the scalar field. This can be achieved by scaling the simulated temperature distribution with field data.

Our observational dataset has been kindly provided by J. Moum from the TOGA-COARE experiment (Fig 1).

We use the spectra of energy (E) and that of fluctuating temperature energy (E_T) to scale the simulated temperature field. The scaling was done in such a way that the ratio of E_T and E from the simulation is set equal to the ratio obtained from the *in situ* measurements for high wavenumbers. (assuming universal equilibrium range spectra). The scaling of the refractive index fluctuations reveals that they are strongly dependent on the observed ratio of ϵ to the rate of dissipation of temperature fluctuations χ. Thus in our simulations we keep ϵ constant ($10^{-6} W/kg$) which fixes the size of the smallest velocity scales. This is among the largest observed dissipation values, which ensures maximum IRI. We then chose two extreme cases of χ: weak ($10^{-11} K^2/s$) and strong ($10^{-6} K^2/s$). These cases are shown in the context of the observational data in Fig 2.

Both values were observed within the surface mixed layer (see Fig 1). The stronger one corresponds to mixing conditions like those at the base of the mixed layer (70 m) while the lower χ is similar to mixed layer values. We have chosen these extreme cases to examine the full range of variability for high ϵ. The high χ value is infrequent: it only made up 1% of the observational data set used here.

We solved equation (1) numerically using the pseudo-spectral second order method described in Strohbehn[1]. The code was written in Fortran for the San Diego Cray C90. The result of simulating light propagation is that we obtain light intensity in W/m^2 and direction in *radians* at cross-sections of the sample cube perpendicular to light direction. Here we report the volume scattering function VSF at a given wavelength ($k = 10^7 m^{-1}$) as the scattered radiant intensity I in a direction θ per unit scattering volume dV divided by the incident irradiance E

$$VSF(\theta) = \frac{dI(\theta)}{E dV}. \tag{2}$$

We also compare the results obtained by propagating light using the geometrical optics method (ray tracing).

4. Results

We determined the spatial distribution of the complex $\psi(x, y)$ of the light after its passage through a simulated turbulent flow field using the parabolized Helmholtz equation. These data were converted into light distribution intensity and the VSF were estimated for each of the two cases of χ.

The emerging from the turbulent flow field irradiance distribution (Fig 3) has a banded structure reminiscent of shadowgraphs taken in laboratory experiments. It is also similar to the observed irradiance distribution obtained using the ray tracing technique[15].

The relative variability of the irradiance $(E - E_0)/E_0$ is of order 10^{-7} in the case of weak χ and 10^{-4} in the case of strong χ. Despite this difference, the spatial distribution of the irradiance is the same. This may be because the Prandtl number is too small, that is the smallest temperature scales are not likely to be generated in the simulation. The smallest IRI are mostly associated with velocity structures; since both cases have the same ϵ, the light distribution would be the same. More importantly, the outgoing light is measured at 20 cm, which is within the pre-caustic (focusing) zone. The relative variability of the irradiance is thus not fully developed[10], as the variability has not reached maximum value. Maximum variability is attained within distance of $O(m)$ from the turbulent volume given the parameters used. It may also be that the outgoing light pattern is independent of χ within a certain parameter range. This is a tantalizing result, but cannot be verified until higher Pr cases are examined beyond the pre-caustic zone, and for a range of ϵ values.

More quantitative results come from analyzing the VSF angle dependence (Fig 4).

Both cases show a similar functional form, which can be divided in three regions. For the smallest angles, the VSF changes very slowly, and there is a threshold beyond which no energy is propagated. From physical reasoning we expect a continuous function. The paucity of points is associated with numerical truncation used in light propagation

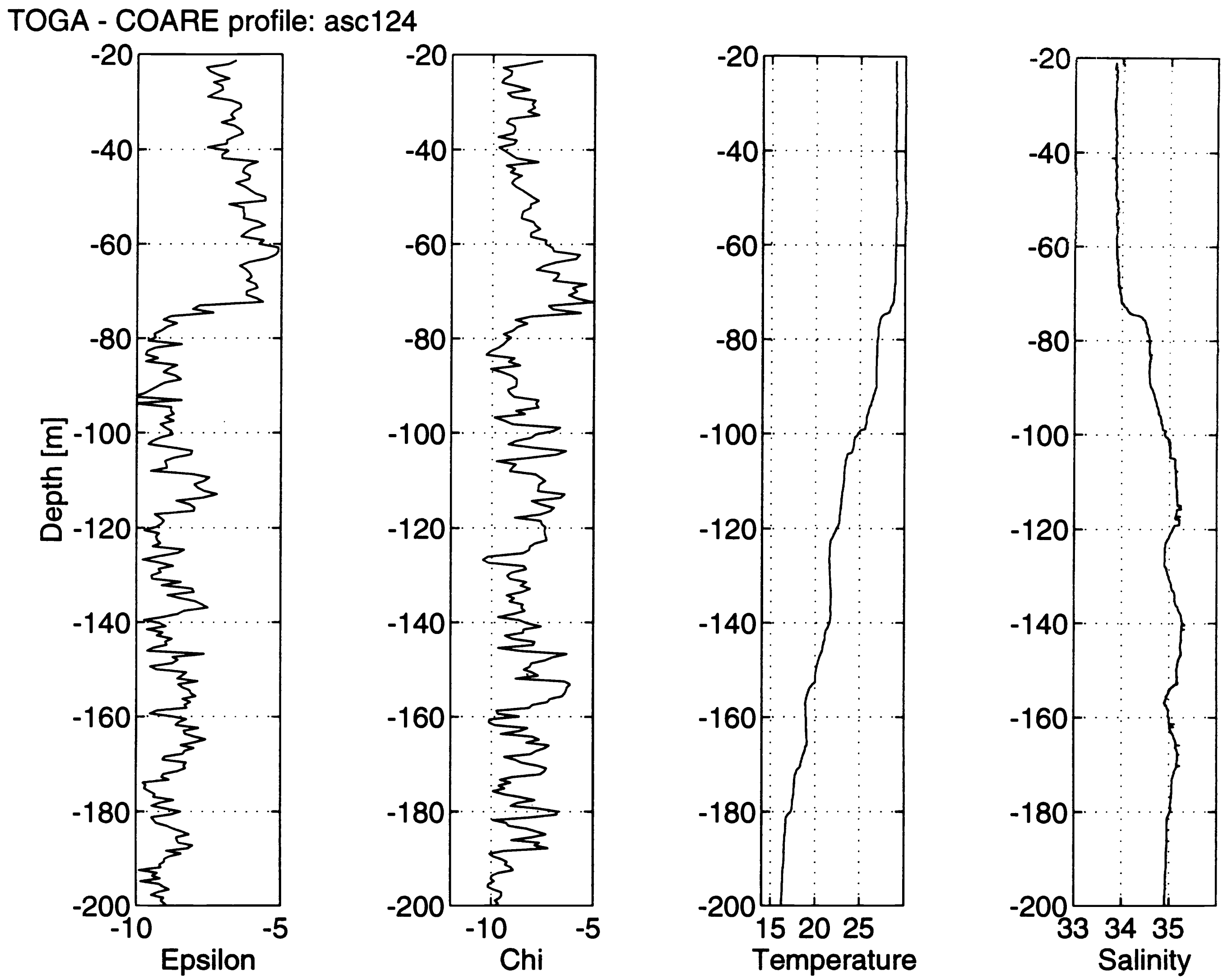
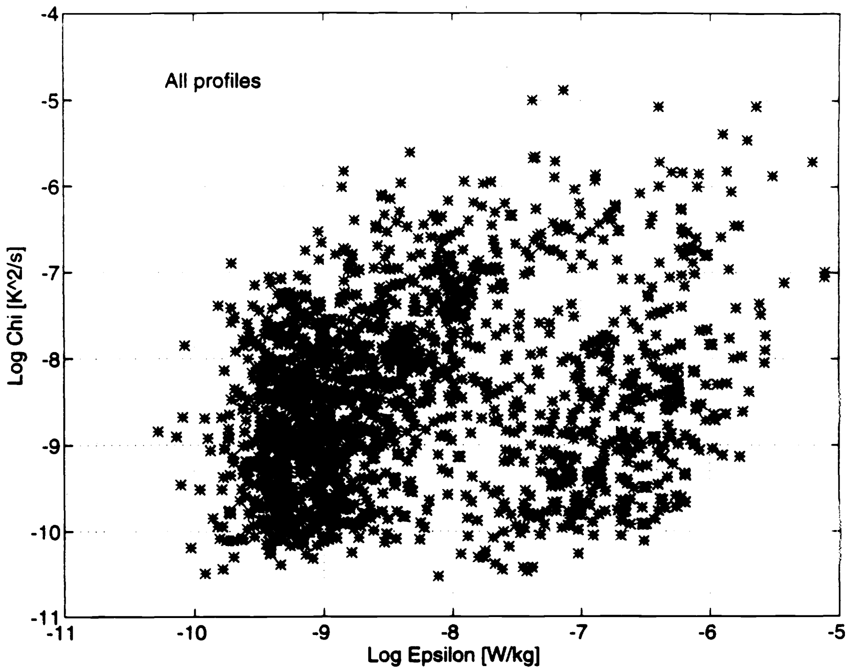


Figure 1:
Profile of ϵ, χ, temperature and salinity from TOGA-COARE experiment (courtessy J. Moum). Our choice of ϵ and χ corresponds to the observed parameters between 60m and 80 m depth



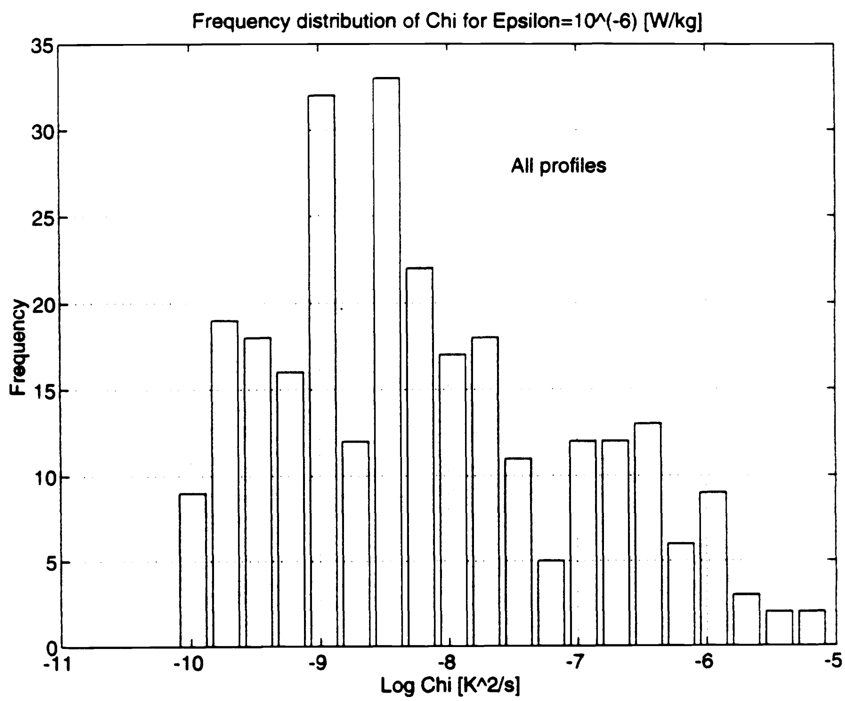


Figure 2:
A. Scatter-plot of the observed ϵ vs. χ in the observational data set. B. Frequency distribution of χ for the chosen $\epsilon = 10^{-6}$ W/kg.

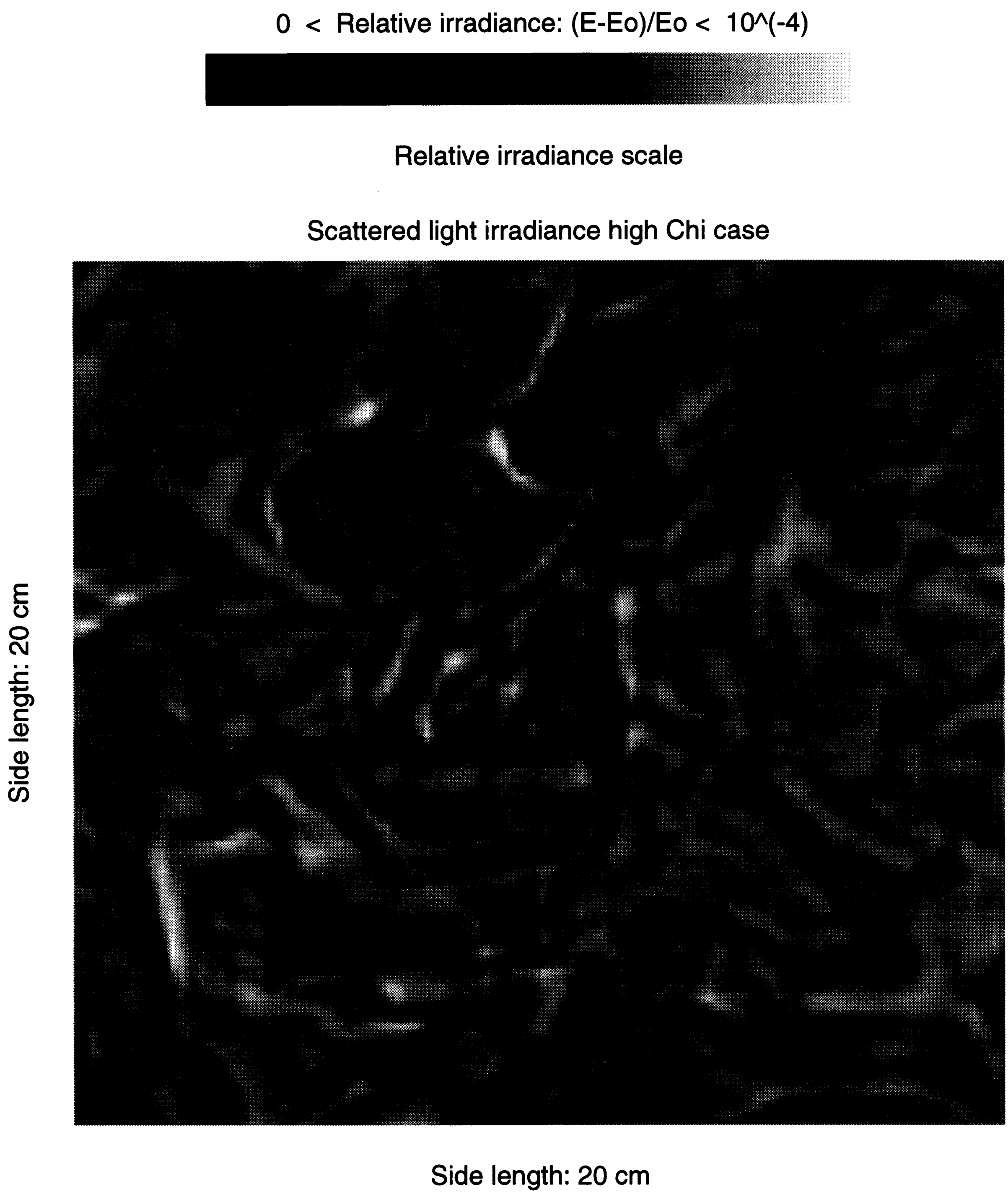


Figure 3:
Spatial distribution of the irradiance after its interaction with the turbulent flow.

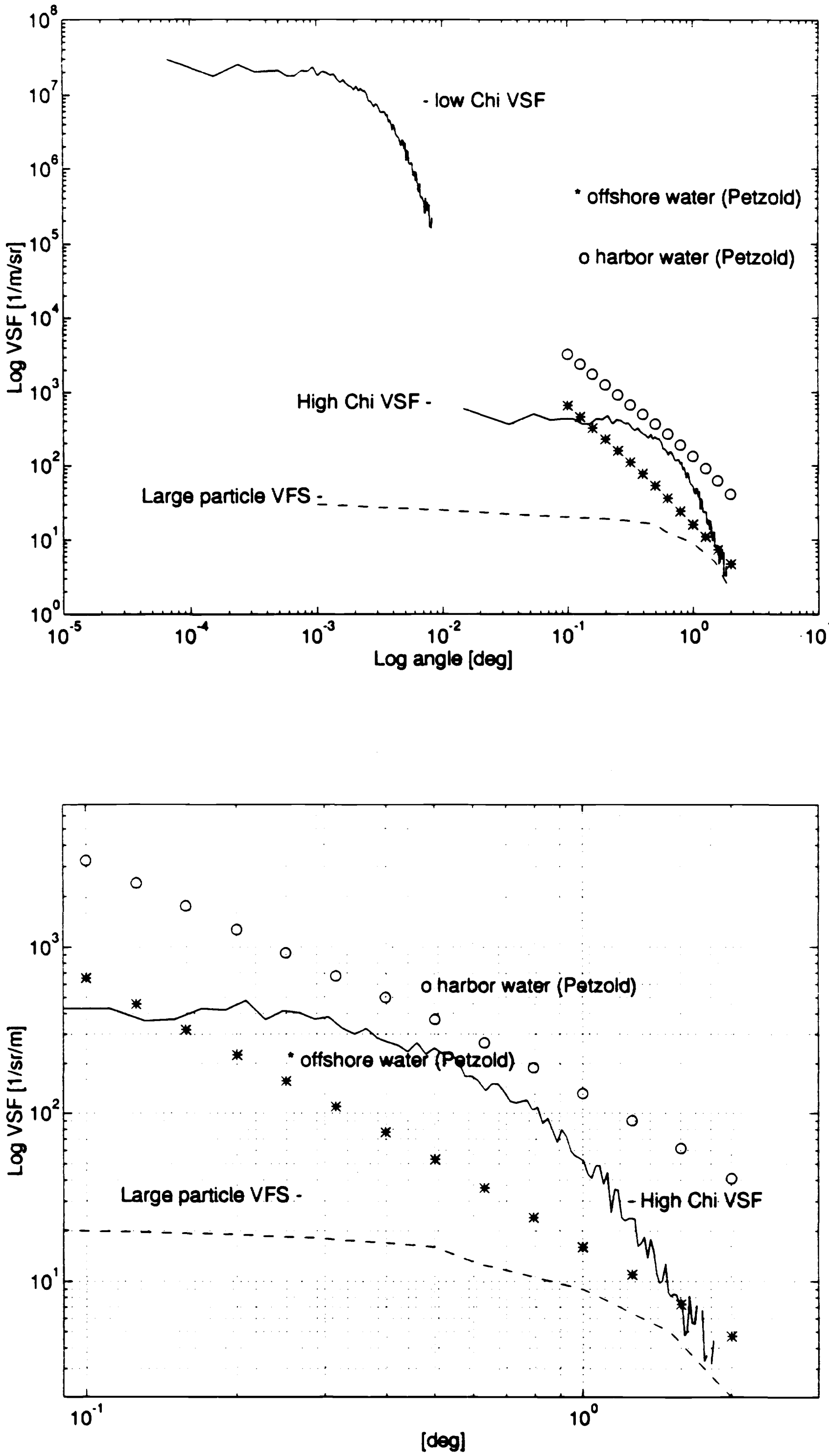


Figure 4:

A. Calculated VSF for two χ cases. VSF measured by Petzold[4] and contribution to VSF from large particles[5] is shown. B. High χ VSF case in log-normal representation with particles contribution to[5] VSF and VSF observed by Petzold[4] routine. For the largest angles, we observe random fluctuations around a constant VSF. This variability is again associated with numerical truncation error. The intermediate region is the main result. This region encompasses two orders of magnitude in scatter angle and three orders of magnitude in VSF. (This is consistent with tests carried out on the numerical solution of equation (1) for a known functions ψ and n, in which the dynamic range was at least three orders of magnitude.) In this region, the VSF decreases exponentially with the scatter angle. This behavior is consistent with the measurements of Petzold[4] and the numerical simulation for Mie scattering on particles carried out by Shifrin[5]. We can observe this in Fig 4. There we can see our results overlayed with the VSF observed by Petzold[4] and compare the particle contribution to the VSF. As we see there the largest particle contribution is fairly constant between angle $0.1°$ and $1°$ and decreases rapidly for angles larger than 1 deg. At the same time for angles between $0.1°$ and $1°$, the observed VSF (Petzold[4]) changes by 2 orders of magnitude similarly to our calculations. These observations were made in the fairly energetic offshore subsurface zone. Our modeled results for the case of high χ show agreement with his field measurements.

In the case of low χ, light is scattered between angles 10^{-4} of $10^{-2}\,°$. For the case of high χ, light is scattered over a greater range to $O(1°)$. Although the simulated high χ case gives scattered light at relatively large angles, this is an infrequent value for the oceanic environment. Nonetheless, uncertainties in optical measurements in seawater due to turbulence-induced scattering may reach up to $1°$ for high temperature variance situations.

The exponential decrease of the VSF with increasing angle was also independently observed in the simulations using the ray tracing technique[15].

5. Conclusions

1. The light intensity distribution exiting the cube exhibits a banded pattern, similar to that observed in laboratory shadowgraphs.

   The pattern obtained by solving the parabolized Helmholtz equation is similar to that obtained from the ray tracing technique.

2. The light intensity distribution for the two cases of χ values are identical. Thus the distribution is probably weakly dependent on the low Pr value and a consequence of sampling within the pre-caustic zone. It may also indicate that there is a parameter space for which light scatter intensity is driven by ϵ only.

3. The volume scattering function is characterized by an exponential decrease of VSF with scatter angle.

4. Our results suggest that turbulence may be responsible for the unexplained scatter at small angles observed in *in situ* experiments.

5. It appears that a scattering angle of $1°$ is the upper limit for turbulence-induced light scatter in the ocean. All measurements relying on light propagating in the ocean within this angle may be affected by turbulence.

6. Acknowledgments

This work was funded by the Ocean Optics Program of the Office of Naval Research under contract number N 00014-94-1-0107. The simulations were carried out at the San Diego Super-computing Center. We thank the support staff for their assistance and efficiency. Dr. James Moum generously provided a very complete suite of oceanic turbulence measurements and Dr. Gregg Ruetsch made his DNS data available, as well as helpful discussion. We are also grateful to Drs. Dariusz Stramski and Mary-Elena Carr for many helpful comments and to Dr. Carr for assistance in manuscript preparation.

7. Literature

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